

An Infinite Impulse Response Equalizer for Magnetic Recording Channels

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Abstract

In this paper, we propose a method of designing an infinite impulse response (IIR) equalizer used to shape the readback signal to a partial response target in longitudinal and perpendicular recording channels. Based on the minimum mean-squared error (MMSE), the coefficients of the IIR equalizer are explicitly derived. For both recording channels, the proposed IIR equalizer performs better than the finite impulse response (FIR) equalizer, especially when the number of equalizer taps is small and the normalized recording density is high.

Introduction

In general, the intersymbol interference (ISI) is a crucial disturbance in magnetic recording channels, especially at high data storage densities. Recently, a read-channel chip architecture employs a finite impulse response (FIR) equalizer to shape the readback signal to a predetermined target before performing maximum-likelihood (ML) equalization by the Viterbi detector (Bergmans,1996) for both in longitudinal and perpendicular recording channels. This technique is known as partial-response maximum-likelihood (PRML) (Cideciyan et al, 1992) which can efficiently combat with ISI.

At high data storage densities, the FIR equalizer with a large number of taps is required because of severe ISI. Nevertheless, the total number of equalizer taps is practically limited by the maximum allowable loop delay in the timing recovery loop (Bergmans,1996) because a small loop delay provides more robust phase locking, which in turn improves the overall system performance.

The partial response (PR) targets of the form $(1-D)(1+D)^n$ and $(1+D)^n$ are suitable for longitudinal and perpendicular recording channels, respectively, where D is the delay operator and n is integer. Given the PR target, its corresponding FIR

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equalizer can then be obtained based on the minimum mean-squared error (MMSE) approach (Moon and Zeng ,1995). In this paper, we propose the design of an infinite impulse response (IIR) equalizer for PR channels.

Many IIR equalizers have been studied in the literature. For instance, IIR modeling was considered in decision feedback equalizer design (Crespo and Honig, 1991) to reduce the number of filter taps. Also, reference (Kim Y. and Moon, J. 1999) investigated the performance of employing continuous-time adaptive IIR equalizers for EPR4 channels. However, in this paper, we directly design the digital IIR equalizer based on the MMSE approach, and then compare its performance with the FIR equalizer.

The rest of this paper is organized as follows. After explaining our channel model in Section II, we describe the design of the IIR equalizers for partial response channels in Section III. Simulation results are provided in Section IV. Eventually, conclusion is given in Section V.

Channel Model

Figure 1 illustrates the channel model for longitudinal and perpendicular recording. A binary input sequence $a_k \in \pm 1$ with bit period T is filtered by an ideal differentiator $1-D$ to form a transition sequence $b_k = (x_k - x_{k-1})/2$, where $b_k = 1$ correspond to a positive or negative transition and $b_k = 0$ corresponds to the absence of a transition. The sequence b_k passes through the channel represented by the transition response $g(t)$. For longitudinal recording, the transition response is a Lorentzian channel, which is defined by (Bergmans, J. W. M., 1996)

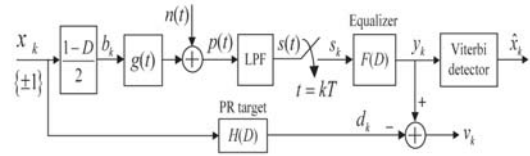


Figure 1. Channel model with equalizer design.

where $\text{erf}(\cdot)$ is an error function defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$
 and PW_{50} is the width of the derivative of $g(t)$ at half of its maximum. We define a normalized recording density (ND) as $ND = PW_{50} / T$, which determines how many data bits can be packed within the resolution unit PW_{50} .

The readback signal $p(t)$ can then be written as

$$p(t) = \sum_{k=0}^{N-1} b_k g(t - kT) + n(t) \tag{1}$$

where N is the length of a data sequence b_k , and $n(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. The readback signal $p(t)$ is filtered by a seventh-order Butterworth low-pass filter (LPF) and is sampled at time $t = kT$, assuming perfect synchronization. The received sequence s_k is equalized such that the output sequence y_k resembles the desired sequence d_k . Finally, the Viterbi detector performs sequence detection to determine the most likely input sequence.

Equalizer Design

The proposed design method can be described by a block diagram as shown in Fig. 2. Assuming that the PR target $H(D)$ is known, meaning that d_k is also known. Let the IIR equalizer $F(D)$ be of the form

$$F(D) = \frac{B(D)}{A(D)} = \frac{b_{-m}D^{-m} + b_{-m+1}D^{-m+1} + \dots + b_0 + b_1D + \dots + b_mD^m}{a_0 + a_1D + a_2D^2 + \dots + a_nD^n}, \quad (4)$$

where a_1, a_2, \dots, a_n and $b_{-m}, b_{-m+1}, \dots, b_0, b_1, \dots, b_m$ are the coefficients of the denominator and the numerator, respectively, which are needed to be optimized. We consider the case where $2m+1$ is less than $n+1$.

From Fig. 2, we can see that

$$s_k * b_k = y_k * a_k \quad (5)$$

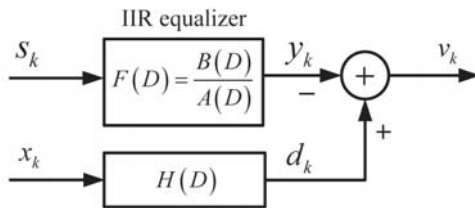


Figure 2. Block diagram for designing an IIR equalizer.

where $*$ denotes the convolution operator.

Substituting $y_k = d_k - v_k$ into (5), we obtain

$$s_k * b_k = (d_k - v_k) * a_k = d_k * a_k - v_k * a_k. \quad (6)$$

We define $v_k * a_k = w_k$ as a filtered error sequence. Thus, (6) can be rewritten as

$$w_k = d_k * a_k - s_k * b_k. \quad (7)$$

Let $\mathbf{A} = [a_0 \ a_1 \ \dots \ a_n]^T$ and $\mathbf{B} = [b_{-m} \ \dots \ b_0 \ \dots \ b_m]^T$ be $(n+1)$ - and $(2m+1)$ -element column vectors, where a_k and b_k denote the coefficients of $A(D)$ and $B(D)$, respectively, and $[\]^T$ represents the matrix transpose operation. Given sequences d_k and y_k , the IIR equalizer can be obtained such that $E\{w_k^2\}$ is minimized in the minimum mean-squared sense, i.e.,

$$E\{w_k^2\} = E\{[(d_k * a_k) - (s_k * b_k)]^2\} \quad (5)$$

where $E\{\square\}$ is the expectation operator.

During the minimization process, we must use the constraint $a_0 = 1$ to avoid reaching the trivial solutions of $A = B = 0$. Therefore, adding a constraint the IIR equalizer is by minimizing (8) subjected to (8) gives

$$E\{w_k^2\} = \mathbf{A}^T \mathbf{R} \mathbf{A} + \mathbf{B}^T \mathbf{S} \mathbf{B} - 2\mathbf{B}^T \mathbf{T} \mathbf{A} - 2\lambda(\mathbf{I}^T \mathbf{A} - 1), \quad (9)$$

where λ is the Lagrange multiplier, \mathbf{I} is an $(n+1)$ -element column vector whose first element is one and the rest is zero.

Taking the derivatives of the right hand side of (9) with respect to \mathbf{B} , \mathbf{A} , and λ , and setting the resulting expressions to zero, one obtain

$$\lambda = \frac{1}{\mathbf{I}^T (\mathbf{R} - \mathbf{T}^T \mathbf{S}^{-1} \mathbf{T})^{-1} \mathbf{I}} \quad (10)$$

$$\mathbf{A} = \lambda (\mathbf{R} - \mathbf{T}^T \mathbf{S}^{-1} \mathbf{T})^{-1} \mathbf{I} \quad (11)$$

$$\mathbf{B} = \mathbf{S}^{-1} \mathbf{T} \mathbf{A} \quad (12)$$

where \mathbf{R} is an $(n+1)$ -by- $(n+1)$ autocorrelation matrix of a sequence d_k , \mathbf{S} is an $(2m+1)$ -by- $(n+1)$ autocorrelation matrix of a sequence s_k , and \mathbf{T} is an $(2m+1)$ -by- $(n+1)$ cross-correlation matrix of sequences s_k and d_k .

Simulation Results and Discussions

We consider the PR target (Tyner and Proakis, 1993) $H(D) = 1 + 2D - 2D^3 - D^4$ for longitudinal recording and $H(D) = 1 + 4D + 6D^2 + 4D^3 + D^4$ for perpendicular recording. The $(2M+1)$ -tap FIR equalizer of the form

$$F_{FIR}(D) = \sum_{k=-M}^M f_k D^k \quad (13)$$

is designed based on the MMSE approach (Moon and Zeng ,1995), which also yields an error sequence v_k that will be used to design the IIR equalizer. The signal-to-noise ratio (SNR) is defined as

$$SNR = 10 \log_{10} \left(\frac{E_i}{N_0} \right) \quad (\text{dB}) \quad (14)$$

where E_i is the energy of the channel impulse response. All equalizers are designed at the SNR required to achieve $BER = 10^{-5}$. Each BER point is computed using as many 4096-bit data sectors as needed to collect 500 error bits, whereas the equalizer taps is designed using only one data sector.

Figure 3 compares the performance of the FIR and IIR equalizers for longitudinal recording as a function of NDs, where “IIR $uZxP$ ” denotes the IIR equalizer with $u = 2m$ zeros (equivalent to $u + 1$ taps) and $x = n$ poles. It is clear from Fig. 3 that when the number of equalizer taps is small (e.g., 3 taps) and ND is high, the IIR equalizer outperforms the FIR equalizer. We also observed that the performance of the IIR and FIR equalizers when the number of taps is large (e.g., more than 5 taps) is similar (not shown here). Furthermore, there is no significant performance improvement by using “IIR $2Z1P$ ” instead of “IIR $2Z2P$.” Thus, it is sufficient to employ the IIR equalizer with one pole in longitudinal recording channels.

We also compare the performance of the IIR and FIR equalizers in perpendicular recording channels as depicted in Fig. 4. Apparently, similar results are obtained as in longitudinal recording. That is, the IIR equalizer performs better than the FIR equalizer, especially when the number of equalizer taps is small and ND is high.

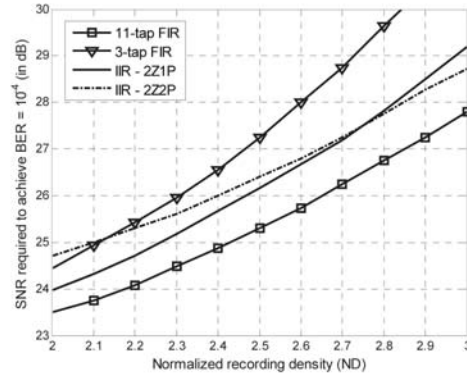


Figure 3. Performance comparison for longitudinal recording.

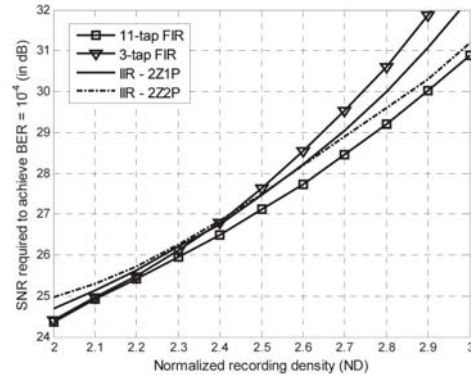


Figure 4. Performance comparison for perpendicular recording.

Again, it is adequate to use employ the IIR equalizer with one pole in perpendicular recording channels.

Generally, the IIR filter has some concern about stability. However, based on our extensive simulations, we have been able to conclude that the proposed IIR equalizer is highly stable for PR channels.

Conclusions

This paper proposed a new IIR equalizer for partial-response channels. For a given PR target, the IIR equalizer can be designed based on the MMSE approach. We found that the proposed IIR equalizer is highly stable for PR channels. When the number of equalizer taps is small (e.g., 3 taps) and the ND is high, the proposed IIR equalizer performs better than the FIR equalizer, especially in longitudinal recording channels. Furthermore, we observed that the IIR equalizer with only one pole is sufficient to be used in magnetic recording channels.

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