On Approximating the Minimum Initial Capital of Fire Insurance with the Finite-time Ruin Probability using a Simulation Approach

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Abstract

This paper considers the discrete time surplus process in the case of fire insurance given by $U_0=u, U_n=U_{n-1}+cZ_n-Y_n$, where $\{Y_n, n \geq 1\}$ is the claim severity process, $\{Z_n, n \geq 1\}$ is the inter-arrival process, c is the premium rate, and $U_0=u \geq 0$ is the initial capital. The claim severities and the inter-arrival time are provided by the Thai Reinsurance Public Co., Ltd. In addition, we assume that $\{Y_n, n \geq 1\}$ and $\{Z_n, n \geq 1\}$ are independent and identically distributed, Y_n has Weibull distribution and Z_n has Poisson distribution. By using the maximum likelihood estimator method, we find that $Y_n \sim$ Weibull (0.8484, 30.5396) and $Z_n \sim$ Poisson (37.8958). Finally, we approximate the finite-time ruin probability for one year by a simulation approach, and use the logarithmic regression to approximate the minimum initial capital corresponding to the quantities of risk $\alpha = 0.01$ and 0.05, respectively.
Keywords: *ruin probability, insurance, minimum initial capital, surplus process*

1. Introduction

In recent years, risk models have attracted much attention in insurance business, in connection with possible insolvencies and capital reserves of insurance companies. From the point of view of any insurance company, the main interest is arrival of claims and their severity; both of which can affect the company's capital. Throughout this paper, we will assume that all random variables are defined in a probability space. The nth claim arriving at time T_n, where $0 = T_0 \leq T_1 \leq T_2 \leq \cdots$, causes the claim severity Y_n. Now, let the constant c be the premium rate for one unit time. The random variable cT_n describes the inflow of capital into the business in time T_n. Thus, $Y_1 + Y_2 + \cdots + Y_n$ describes the outflow of capital due to payments for claims occurring up to time T_n. Therefore, the quantity

$$U_n = u + cT_n - \sum_{i=1}^{n} Y_i \qquad (1.1)$$

is the insurance's balance (or surplus) at time T_n for n=1,2,3,…, where the constant $u \geq 0$ is the initial capital. For convenience,

we define the inter-arrival time by $Z_n = T_n - T_{n-1}$, which is the length of time between $(n-1)^{th}$ claim and the nth claim. Thus, the process becomes

$$U_0 = u, \qquad U_n = U_{n-1} + cZ_n - Y_n. \qquad (1.2)$$

In this paper, we consider the quantity of risk in an insurance company up to a given time T corresponding to the initial capital u. The probability of the first time that the surplus becomes negative up to time corresponding to the initial capital u is called the finite-time ruin probability, which is defined by

$$\varphi(u, c, T) = \Pr(U_n < 0, \text{for some } n = 1,2,3, \ldots \text{ and } T_n \leq T | U_0 = u). \qquad (1.3)$$

The general approach for studying ruin probability in the discrete time surplus process is via the so-called Gerber-Shiu discounted penalty function; see for examples, Dickson(1), Li (2) and Pavlova and Willmot (4). All of these articles studied the ruin probability as a function of the initial capital u. In 2013, Sattayatham et al. (5) studied in the opposite direction, i.e., they considered the minimum initial capital (MIC) to ensure that the ruin probability did not exceed a given quality α and the given premium rate c. ; this is called the ruin-probability-based initial capital problem of the discrete-time surplus process. In this paper, we shall study the MIC with corresponding to the quality α as a function of premium rate c by using a simulation approach.

2. Materials and methods

The claim severity process $\{Y_n, n \geq 1\}$ and the inter-arrival time process $\{Z_n, n \geq 1\}$ are defined as mentioned in the model (1.1) and assumed to be independent and identically distributed, in addition, $\{Y_n, n \geq 1\}$ and $\{Z_n, n \geq 1\}$ are assume to be mutually independent.

Fire insurance is one of non-life insurances that happened infrequently, but has high severity. Usually, the distribution of the claim severities due to fire accidents is assumed to be heavy tails. We found that the Weibull distribution is a heavy tails if the shape parameter, a, is less than one and the scale parameter, b, is greater than zero. In particular, we consider the claim severities that are greater than 20 million Baht. In this study, we therefore consider the location parameter $\gamma = 20$ and assume that the claim severities, Y_n, has a Weibull distribution with the two-parameters (2P). The probability density function for Y_n is given by

$$f(x; a, b) = \begin{cases} \frac{a}{b}\left(\frac{x-20}{b}\right)^{a-1} \exp\left(-\left(\frac{x-20}{b}\right)^a\right), & x \geq 20 \\ 0, & x < 20 \end{cases} \qquad (2.1)$$

where $a > 0$ and $b > 0$ are the shape and the scale parameters, respectively. The parameter vector (a,b) is estimated by using the maximum likelihood estimator (MLE) method. The estimated parameter is obtained by

$$b = \left(\frac{\sum_{i=1}^{n}(x_i - 20)^a}{n}\right)^{\frac{1}{a}} \qquad (2.2)$$

and

$$\sum_{i=1}^{n} \ln(x_i - 20) + \frac{n}{a} - n \ln b - \sum_{i=1}^{n}\left(\frac{x_i - 20}{b}\right)^a \ln\frac{x_i - 20}{b} = 0. \qquad (2.3)$$

In this research we approximate a, b by using the bisection method with initial points a=0.05 and 1.0. Next, we assume that the inter-arrival time, Z_n, has a Poisson distribution with the (mean) parameter λ. We estimate the parameter with the MLE method and obtain the estimated formula as

$$\lambda = \frac{1}{n}\sum_{i=1}^{n} z_i, \qquad (2.4)$$

where z_i is the ith inter-arrival claim time.

3. Results and discussion

The data of the claim severities of fire insurance in Thailand from 2000 to 2004 provided by the Thai Reinsurance Public Co., Ltd. is considered. The data consist of the claim severities and the claim times shown as the total claim in Figure 1.
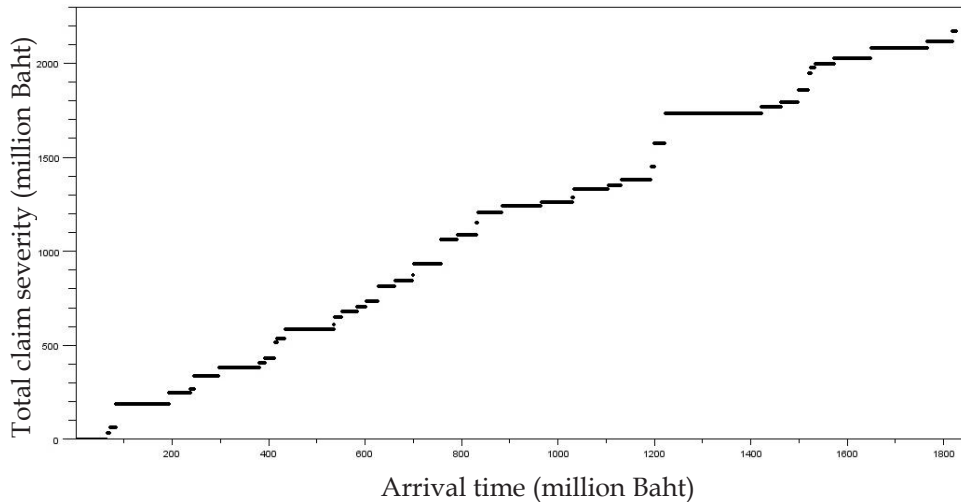

Figure 1. Claim severities and claim times

Using the MLE method, we find that the estimated parameter vector of the claim severities is (a,b) = (0.8484,30.5396) for Weibull distribution, and the estimated parameter of the inter-arrival time is also found to be λ=37.8958 for Poisson distribution.

The simulation result is carried out with 100,000 paths for the surplus process as mentioned in the surplus process (1.1). We assume that Y_n~Weibull (0.8484,30.5396) and Z_n~Poisson (37.8958) for all *n*. The simulation results of finite-time ruin probabilities with the several initial capital and premium rate are shown in Figure 2.
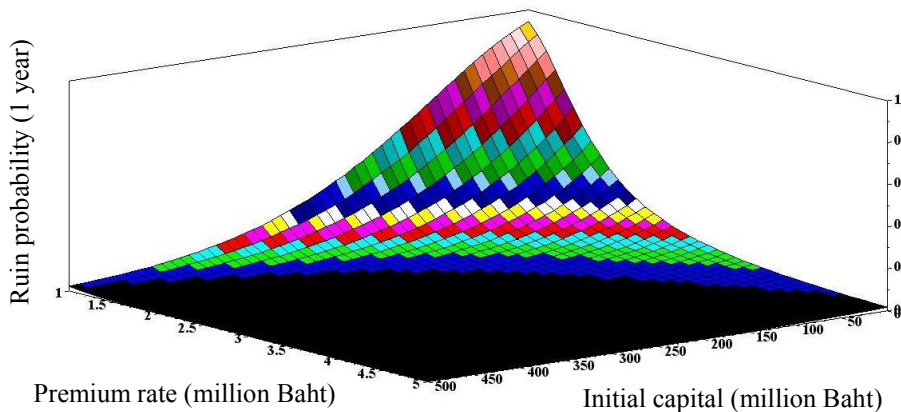

Figure 2. The approximation of the finite-time ruin probability

Figure 2 shows the approximation of the finite-time ruin probability of the surplus process (1.1) with Weibull distributed claims and Poisson distributed inter-arrival times with 100,000 paths corresponding to the initial capital u and the premium rate *c* when u = 0,10,20,…,500 million Baht and c = 1.0,1.1,.1.2,…,5.1 million Baht.

From the simulation result, for each premium rate c = 1.0,1.1,.1.2,…,5.1, we can choose the initial capital u which can control the finite time ruin to be not greater than the quantity of the finite-time ruin probability α = 0.01 and α=0.05. The result is shown in Table 1.

Table 1. Simulation results of the MIC (million Baht)

c	MIC		c	MIC		c	MIC	
	α=0.01	α=0.05		α=0.01	α=0.05		α=0.01	α=0.05
1.0	590	430	2.4	270	120	3.8	150	0
1.1	550	390	2.5	260	110	3.9	130	0
1.2	520	360	2.6	240	100	4.0	130	0
1.3	490	330	2.7	240	90	4.1	120	0
1.4	460	300	2.8	220	80	4.2	110	0
1.5	420	270	2.9	220	70	4.3	110	0
1.6	400	250	3.0	210	60	4.4	100	0
1.7	380	230	3.1	200	50	4.5	90	0
1.8	360	210	3.2	190	40	4.6	90	0
1.9	340	190	3.3	190	40	4.7	80	0
2.0	320	170	3.4	180	30	4.8	70	0
2.1	300	160	3.5	170	20	4.9	70	0
2.2	300	140	3.6	160	10	5.0	60	0
2.3	280	130	3.7	150	10	5.1	50	0

Table 1 shows the chosen initial capitals u corresponding to the given premium rates c, which are represented as the approximation of the MIC to control the finite time ruin probability to be greater than the quantity α = 0.01 and 0.05 of the surplus process (1.1).

In this paper, we consider the premium rate c = 1.0,1.1,1.2,…, 5.1 and apply the logarithmic regression to approximate the MIC corresponding to other premium rates. The obtained results are shown in Figure 3.
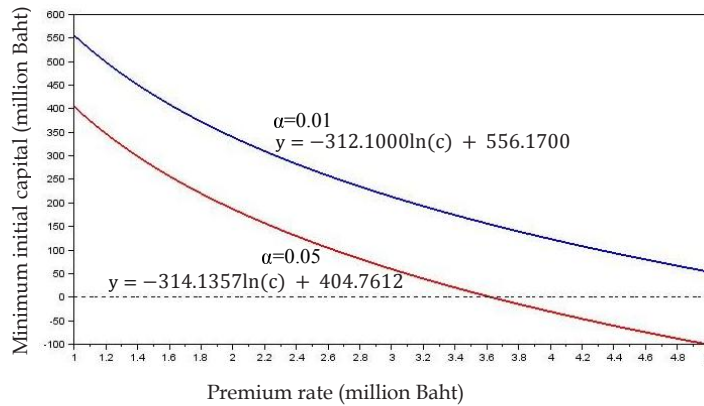
Figure 3. Logarithmic regression of the MIC (α = 0.01 and 0.05) and the premium rates

Figure 3 shows the relation between the premium rate and the MIC. The upper line represents the case of α=0.01, which has the MIC function

$$u^*(c; 0.01) = \max\{0, -312.1000\ln(c) + 556.1700\}. \quad (3.1)$$

The lower line in figure 3 represents the case of α = 0.05, which has the MIC function

$$u^*(c; 0.05) = \max\{0, -314.1357\ln(c) + 404.7612\}. \quad (3.2)$$

4. Applications

Assume that an insurance company compute the premium rate by using the expected value premium principle (3) with safety loading θ = 3.50, i.e.,

$$c = (1 + \theta) \cdot \frac{E[Y]}{E[Z]} = (2.75) \cdot \frac{b\Gamma\left(1 + \frac{1}{a}\right)}{\lambda} = 3.9501. \quad (4.1)$$

Therefore, we obtain the minimum initial capital as $u^*(3.9501; 0.01) = \max\{0, 127.43\} = 127.43$ and $u^*(3.9501; 0.05) = \max\{0, -26.77\} = 0$. This means that the insura nce company has toreserve at least 127.43 million Baht for the initial capital against the insolvency according as the level of confidence 99%, while the company does not need to reserve the initial capital for the level of confidence 95%.

5. References

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